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Energy Relations in Natural and Artificial Diamagnetic Materials

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Final Performance Report for Grant FA9550-13-1-0033: "Energy Relations in Natural and Artificial Diamagnetic Materials"

Program Manager: Dr. Arje Nachman,   Principal Investigator: Dr. Arthur D. Yaghjian

1 Introduction

As a consequence of the insight gained from the rigorous homogenization theory developed by the principal investigator in collaboration with Andrea Alù and Mário Silveirinha [1] for the magnetic permeability as well as electric permittivity of 3D spatially dispersive periodic arrays, the AFOSR three-year Grant FA9550-13-1-0033 for $110,889 and its Additional Work Effort for $72,600 was awarded to S4-Inc to have Dr. Yaghjian apply this theory to determine consistent expressions for the power and energy densities in a diamagnetic continuum. The tasks proposed and successfully completed for the three years were in brief:

1. To evaluate the external and internal power and energy relations for a realistic model of a diamagnetic molecule or inclusion, such as a perfectly electrically conducting (PEC) wire loop;
2. To determine a reliable method for obtaining macroscopic continuum relations of power and energy from the microscopic Maxwellian equations describing the power and energy of diamagnetic molecules or inclusions.
3. To incorporate the results for the macroscopic power and energy relations into the continuum formulation for diamagnetic materials or metamaterials;
4. To combine the diamagnetic relations for power and energy with those of electric polarization to obtain positive semi-definite (that is, nonnegative) energy expressions for diamagnetic continua, then document the results of the research in archival publications.

These accomplishments, which were recently submitted to the journal *Physical Review B*, will be summarized in the remainder of this final report. Some of the results can also be found in the references [2]–[4].

There are several reasons for determining nonnegative macroscopic energies from the microscopic Maxwellian equations for classical diamagnetic dipolar continua. First, it is one of the remaining unresolved problems in classical electromagnetic theory even though the problem is relatively easy to state: given a volume of a macroscopic continuum satisfying Maxwell's dipolar equations and illuminated by external fields, are there macroscopic energy densities that never become negative for all times after an initial time when the macroscopic fields and polarizations are zero. Nonnegative energy expressions provide a means for determining realistic physical limitations, such as the upper bounds on the bandwidth and gain of antennas or the lower bounds on antenna quality factors. Valuable inequalities satisfied by bulk constitutive parameters as well as by the group and energy transport velocities in materials and metamaterials can be obtained from nonnegative macroscopic energy expressions. Also, knowing the sufficient conditions for the validity of the nonnegative energy expressions lends insight into the development and use of materials and metamaterials that may not be subject to the restrictions imposed by these expressions.

2 Energy Supplied to the Charge Carriers

The energy supplied to the charge carriers (current density $\mathbf{j}$) in a material volume V by the electromagnetic fields (electric field $\mathbf{e}$) during the time interval $t - t_0$ is given by

$$W_{je}(t) - W_{je}(t_0) = \int_{t_0}^{t} \int_V \mathbf{j}(\mathbf{r}, t) \cdot \mathbf{e}(\mathbf{r}, t)\, dV\, dt\,. \tag{1}$$

Assume that the applied electromagnetic fields are zero until after the time t_0 so that the in-band electric field does no work on the charge carriers until after t_0. If the charge-current is generated by bound charge carriers whose only net energy transfer between them and their host material is in the form of heat (out-of-band energy), then this work, $W_{je}(t) - W_{je}(t_0)$, done on the bound charge carriers equals the change in in-band kinetic and potential energy of the charge carriers plus any energy converted into heat. Moreover, if the macroscopic continuum is passive in that there are no auxiliary sources (such as capacitive energy stored in a compressed-spring model of electric dipoles formed by equal and opposite electric charges, or inductive energy stored in permanent molecular Amperian magnetic dipoles) *that can release energy* upon excitation by applied fields to serve as an active source of internal energy adding to the in-band energy of the charge carriers, then $W_{je}(t) - W_{je}(t_0) \geq 0$. Choosing the arbitrary, constant initial energy $W_{je}(t_0)$ equal to zero, we have

$$W_{je}(t) = \int_{t_0}^{t} \int_V \mathbf{j}(\mathbf{r}, t) \cdot \mathbf{e}(\mathbf{r}, t)\, dV\, dt \geq 0 \tag{2}$$

for bound charge carriers in passive material that can be lossy as well as lossless. We will refer to a material that satisfies the inequality in (2) for all V as being *unconditionally passive*. The current $\mathbf{j}(\mathbf{r}, t)$ in (2) can include conduction current as well as the current produced by the local motion of the bound charges if the changes in the kinetic and potential energy transferred by the drift velocity of the conduction charges are negligible compared with the out-of-band heat energy produced by the conduction current (as is normally the case).

3 Poynting's Theorem for Dipolar Continua

It can be proven that the total electromagnetic power at time t entering a closed surface S lying in free space is given by the integral over S of the microscopic Poynting vector, namely

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{e}(\mathbf{r}, t) \times \mathbf{h}(\mathbf{r}, t)]\, dS \tag{3}$$

where $\hat{\mathbf{n}}$ is the unit normal directed away from the volume V enclosed by S, and the microscopic magnetic fields are related by $\mathbf{b} = \mu_0 \mathbf{h}$ because S is in free-space.

From Maxwell's equations in an ideal dipolar continuum, it follows that the components of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$, tangential to an interface between free space and the polarized material are continuous, provided there are no equivalent polarization surface currents at the interface of the polarized material, that is, no delta functions in the electric and magnetic polarization densities, $\mathbf{P}$ or $\mathbf{M}$, at the interface [5]. Such delta functions can occur for polarization with constitutive parameters that are strongly spatially dispersive [1] and even for spatially nondispersive material characterized by extreme anisotropic constitutive parameters with the values of some elements approaching zero and others infinity at the in-

terface [5]. If we avoid these pathologically extreme constitutive parameters and also assume low enough

frequencies that spatial dispersion is negligible, then the tangential components of the ideal-continuum $\mathbf{E}$ and $\mathbf{H}$ are continuous across the free-space/dipolar-material interface. This implies that if we remove an infinitesimally thin shell of the ideal continuum containing S (everywhere that the closed surface S passes through the dipolar continuum), the value of $\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})$ will be continuous across the resulting free-space shell. Moreover, since the continuum fields equal the microscopic fields in the free-space shell, that is

$$\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H}) = \hat{\mathbf{n}} \cdot (\mathbf{e} \times \mathbf{h}) \tag{4}$$

in the free-space shell, one finds from (3) that the total instantaneous power flowing across a closed surface S in an ideal dipolar continuum is given by the integration of the continuum Poynting vector $P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)]dS$, or with the aid of Maxwell's ideal-continuum dipolar equations

$$P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)]dS = \int_V \left[\frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} \cdot \mathbf{E}(\mathbf{r}, t) + \frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \cdot \mathbf{H}(\mathbf{r}, t)\right] dV\,. \tag{5}$$

Thus, we have determined that in an ideal continuum the integral of the continuum Poynting vector over a closed surface S of a volume V exactly equals the instantaneous power flow across that surface, provided the constitutive parameters do not take on extreme enough values and the spatial dispersion is not strong enough over the operational bandwidths to produce equivalent surface polarization currents at a hypothetical interface S between the polarized material and free space. Furthermore, the surface integral of the continuum Poynting vector equals the corresponding volume integral of the fields on the right-hand side of (5).

It should be noted that in some contexts bianisotropic media are considered to be spatially dispersive because the electric and magnetic fields in bianisotropic media are proportional to the curls of the magnetic and electric fields, respectively. However, since this weak spatial dispersion maintains continuous $\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})$ across interfaces between the bianisotropic continua and free space, for the purposes of this work, we can include the weak spatial dispersion of bianisotropic media within the category of spatially nondispersive media.

The foregoing argument that has led to Poynting's theorem in an ideal dipolar continuum and its interpretation in terms of power flow can be applied with a couple of modifications to a macroscopic dipolar continuum comprised of discrete electric and magnetic dipoles. The first modification is that the volume V with surface S to which Poynting's theorem is applied should be no smaller than a macroscopic volume ΔV which is electrically small but still contains so many dipoles that the averaging of the fields over ΔV gives to a good approximation the continuum fields satisfying Maxwell's equations.

The second modification involves a transition layer of finite thickness at an interface between free space and the macroscopic dipolar continuum [1], [6]. Unlike an ideal dipolar continuum, the tangential $\mathbf{E}$ and $\mathbf{H}$ fields in a macroscopic dipolar continuum are not perfectly continuous across an interface but are nearly continuous across a transition layer of thickness δ containing the interface. Fortunately, under the conditions that the discrete dipolar material behaves as a continuum, analytical and numerical results with discrete dipolar arrays indicate that the thickness δ of the transition layer is on the order of the average separation distance d of the dipoles [7], [8]. This means that the infinitesimal thickness of the free-space shell about S that was chosen to derive the above results for the Poynting vectors for the ideal continuum can be made approximately equal to the thickness δ of the transition layer without appreciably changing the average fields within the volume V. Consequently, the equation (4) holds to a good approximation for S at the center of the δ-thick free-space shell that removes a discrete number of dipoles; see Fig. 1. The better the discrete distribution of dipoles approximates a continuum, the thinner the transition layer becomes ($\delta \rightarrow 0$) and

the more accurate is the approximation for the macroscopic continuum. The only precedent that we could find for this distinction between ideal and macroscopic continua was in Maxwell's original Treatise. This discovery became one of the highlights of a 2014 paper invited to a special issue of the journal, Progress in Electromagnetics Research, commemorating 150 years of Maxwell's equations [2], [3].

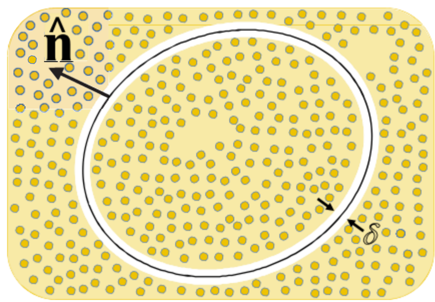

Figure 1: Volume V with its surface S centered in a free-space shell of thickness δ enclosing a large number of discrete electric and magnetic dipoles.

The most important aspect of (5) is that both the surface and volume integrals in (5) are equal to the instantaneous power flow $P(t)$ through the closed surface S into the volume V, exactly for the ideal continuum and approximately for the macroscopic continuum, respectively. Although free-space shells are invoked to prove (5), the fields in (5) are the continuum fields and thus (5) holds exactly within the ideal continuum, and approximately in the macroscopic continuum since $\hat{\mathbf{n}}\cdot(\mathbf{E}\times\mathbf{H})$ is continuous across a free-space/dipolar-material interface under the conditions specified above and satisfied by most dipolar metamaterial continua as well as natural dipolar material continua.

4 Energy Relations for Dipolar Continua

If the volume V is chosen to be a macroscopic volume ΔV that is electrically small but large enough to contain a great number of discrete dipoles, then (5) becomes

$$\Delta P(t) \approx -\oint_{\Delta S} \hat{\mathbf{n}}\cdot[\mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t)]dS = \int_{\Delta V}\left(\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}\cdot\mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}\cdot\mathbf{H}(\mathbf{r},t)\right)dV\,. \quad (6)$$

We can also express the power $\Delta P(t)$ in terms of the microscopic fields and sources within ΔV. This can be done by reinstating the surface ΔS of ΔV in the middle of a free-space shell of transition layer thickness δ that removes a discrete number of dipoles, as explained in Section 3, where the volume ΔV contains a large number of discrete electric and magnetic dipoles. Since ΔS is now in free space, the average macroscopic $\mathbf{E}$ and $\mathbf{H}$ fields there are equal to the microscopic $\mathbf{e}$ and $\mathbf{h} = \mathbf{b}/\mu_0$ fields, as explained in Section 3. Thus, combining the macroscopic Poynting's theorem in (6) with the corresponding microscopic Poynting's theorem by means of the equality of the macroscopic and microscopic Poynting's vectors in (4), it can be shown that the following relationship and inequality hold for *unconditionally passive* material

$$\int_{t_0}^{t}\left(\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{M}(\mathbf{r},t')\right)dt' \approx \frac{1}{\Delta V}\int_{t_0}^{t}\int_{\Delta V}\mathbf{j}\cdot\mathbf{e}\,dV\,dt'$$

$$+\frac{1}{2\Delta V}\int_{\Delta V}\left\{\epsilon_0|\mathbf{e}^{\rm ins}(\mathbf{r},t)-\mathbf{E}^{\rm ins}(\mathbf{r},t)|^2+\frac{1}{\mu_0}|\mathbf{b}^{\rm ins}(\mathbf{r},t)-\mathbf{B}^{\rm ins}(\mathbf{r},t)|^2\right\}dV \geq 0 \tag{7}$$

where the superscripts "ins" refer to the microscopic and macroscopic fields produced only by the microscopic and macroscopic dipolar sources inside ΔV, respectively.

A dipolar material is not necessarily unconditionally passive if it contains permanent dipole moments whose magnitudes can be reduced when subject to applied fields to create an effectively active material. However, nearly all electric polarization can be modeled by either initially randomly oriented molecules with permanent electric dipole moments that stay practically fixed in magnitude as they align in the applied field, or by initially zero electric dipole-moment molecules that distort in the applied field to produce induced electric dipole moments [9, p. 464], [10, p. 162]. In the latter case, the electric dipoles have no initial energy from which to draw. In the former case, the energy of formation of the electric dipoles cannot be reduced because the initial separation distance between the equal and opposite charges of each dipole cannot increase to allow their mutual force to do negative work and release part of their initial energy of formation. Thus, these two types of electric dipoles satisfy the criterion of unconditional passivity.

4.1 Macroscopic Polarization Energy for Magnetic Dipoles

Since magnetic charge does not exist, magnetic dipoles are created by molecules with circulating currents that can be modeled by perfectly electrically conducting (PEC) wire loops. If the wire loops carry no permanent current so that all the circulating current and magnetic dipole moments are induced by the applied fields, then the distribution of molecules forms a diamagnetic macroscopic continuum whose low-frequency magnetic susceptibility is less than zero.

4.1.1 Macroscopic Polarization Energy for Diamagnetism

In the case of diamagnetism, the initial value of the current and magnetic dipole moment of the wire loops are zero. Therefore, diamagnetic-material continua that can also contain electric polarization produced by electric dipoles with fixed-magnitude or initially zero electric dipole moments exhibit the property of unconditional passivity and thus diamagnetic material satisfies the inequality in (7), specifically

$$\int_{t_0}^{t}\left[\frac{\partial\mathbf{P}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{E}(\mathbf{r},t')-\frac{\partial\mathbf{B}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{M}(\mathbf{r},t')\right]dt' \geq 0\,. \tag{8}$$

As far as we are aware, this is the first consistent, positive semi-definite energy expression that has been derived for macroscopic diamagnetic material.

As a simple example, consider a material with negligible loss and constant real permittivity ϵ and permeability μ over the operational baseband bandwidth $|\omega| < \omega_0$. Then $\mathbf{P} = (\epsilon-\epsilon_0)\mathbf{E}$ and $\mathbf{M} = (1/\mu_0 - 1/\mu)\mathbf{B}$ over the baseband bandwidth, and (8) reduces to

$$(\epsilon-\epsilon_0)|\mathbf{E}|^2-(1/\mu_0-1/\mu)|\mathbf{B}|^2 \geq 0\,. \tag{9}$$

Since $\mathbf{E}$ and $\mathbf{B}$ can be chosen equal to zero independently, this inequality reveals that

$$\epsilon \geq \epsilon_0\,, \quad 0 \leq \mu \leq \mu_0 \tag{10}$$

which confirms that the inequality in (8) applies to diamagnetic continua.

5 Application of Energy Relations to Bianisotropic Continua

In this section, the energy theorems summarized immediately above are applied to linear, passive, spatially nondispersive media to obtain frequency-domain expressions for internal energy densities in lossless media and inequalities that the linear constitutive relations must obey in lossless media. The most general linear, spatially nondispersive constitutive relations are those for bianisotropic media and are given in the frequency domain as

$$\mathrm{D}_\omega(\mathrm{r}) = \bar{\bar{\epsilon}}(\mathrm{r})\cdot\mathrm{E}_\omega(\mathrm{r}) + \bar{\bar{\tau}}(\mathrm{r})\cdot\mathrm{H}_\omega(\mathrm{r}) \tag{11a}$$

$$\mathrm{B}_\omega(\mathrm{r}) = \bar{\bar{\mu}}(\mathrm{r})\cdot\mathrm{H}_\omega(\mathrm{r}) + \bar{\bar{\nu}}(\mathrm{r})\cdot\mathrm{E}_\omega(\mathrm{r}) \tag{11b}$$

where $\bar{\bar{\mu}}(\mathrm{r})$, $\bar{\bar{\epsilon}}(\mathrm{r})$, and $[\bar{\bar{\nu}}(\mathrm{r}), \bar{\bar{\tau}}(\mathrm{r})]$ are the permeability dyadic, the permittivity dyadic, and the magneto-electric dyadics, respectively. Like the fields, they are, in general, functions of frequency ω and position r within the media.

If the magnetic polarization of the bianisotropic material is produced by diamagnetic dipoles, then the inequality in (8) applies. It can be rewritten as

$$\int_{t_0}^{t}\left[\frac{\partial \mathrm{D}(\mathrm{r},t')}{\partial t'}\cdot\mathrm{E}(\mathrm{r},t') + \frac{\partial \mathrm{B}(\mathrm{r},t')}{\partial t'}\cdot\mathrm{H}(\mathrm{r},t')\right]dt' \ge \frac{1}{2}\left[\epsilon_0|\mathrm{E}(\mathrm{r},t)|^2 + \frac{1}{\mu_0}|\mathrm{B}(\mathrm{r},t)|^2\right]. \tag{12}$$

Using an analysis similar to the one in [11] and [12] but applied to (12), we find for a lossless frequency window

$$\mathrm{Re}\left\{\mathrm{E}^*_\omega\cdot(\omega\bar{\bar{\epsilon}})'\cdot\mathrm{E}_\omega + \mathrm{H}^*_\omega\cdot(\omega\bar{\bar{\mu}})'\cdot\mathrm{H}_\omega + \mathrm{E}_\omega\cdot\left[\omega(\bar{\bar{\nu}}^{\mathrm{T}} + \bar{\bar{\tau}}^*)\right]'\cdot\mathrm{H}^*_\omega\right\} \ge \left[\epsilon_0|\mathrm{E}_\omega|^2 + \frac{1}{\mu_0}|\mathrm{B}_\omega|^2\right] \tag{13}$$

which inplies that

$$[(\omega\epsilon_{ll})' - \epsilon_0] \ge \omega\epsilon'_{ll}/2 \ge 0 \tag{14a}$$

with a similar (though not identical) inequality for the diagonal elements of the permeability dyadic if the orientation of the xyz coordinate system is chosen at each point in space to make the permeability dyadic diagonal (which is always possible because the lossless permeability dyadic is Hermitian), namely

$$[(\omega\mu_{ll})' - \mu_{ll}^2/\mu_0] \ge \omega\mu'_{ll}/2 \ge 0\,. \tag{14b}$$

Note that as $\omega\to 0$, (14a) implies that

$$\epsilon_{ll}(\omega\to 0) - \epsilon_0 \ge 0 \tag{15}$$

(assuming $\lim_{\omega\to 0}(\omega\epsilon'_{ll}) = 0$). However, (14b) implies that

$$\mu_{ll}(\omega\to 0)[1 - \mu_{ll}(\omega\to 0)/\mu_0] \ge 0 \tag{16}$$

or, equivalently

$$0 \le \mu_{ll}(\omega\to 0) \le \mu_0 \tag{17}$$

for diamagnetic material (assuming $\lim_{\omega\to 0}(\omega\mu'_{ll}) = 0$). These inequalities are compatible with diamagnetic material, yet incompatible with ordinary nondiamagnetic (paramagnetic) material.

Acknowledgment

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The energy supplied to the charge carriers (current density $\mathbf{j}$) in a material volume V by the electromagnetic fields (electric field $\mathbf{e}$) during the time interval $t - t_0$ is given by

$$W_{je}(t) - W_{je}(t_0) = \int_{t_0}^{t} \int_{V} \mathbf{j}(\mathbf{r}, t') \cdot \mathbf{e}(\mathbf{r}, t') dV dt' \,. \tag{1}$$

Assume that the applied electromagnetic fields are zero until after the time t_0 so that the in-band electric field does no work on the charge carriers until after t_0. If the charge-current is generated by bound charge carriers whose only net energy transfer between them and their host material is in the form of heat (out-of-band energy), then this work, $W_{je}(t) - W_{je}(t_0)$, done on the bound charge carriers equals the change in in-band kinetic and potential energy of the charge carriers plus any energy converted into heat. Moreover, if the macroscopic continuum is passive in that there are no auxiliary sources (such as capacitive energy stored in a compressed-spring model of electric dipoles formed by equal and opposite electric charges, or inductive energy stored in permanent molecular Amperian magnetic dipoles) *that can release energy* upon excitation by applied fields to serve as an active source of internal energy adding to the in-band energy of the charge carriers, then $W_{je}(t) - W_{je}(t_0) \geq 0$. Choosing the arbitrary, constant initial energy $W_{je}(t_0)$ equal to zero, we have

$$W_{je}(t) = \int_{t_0}^{t} \int_{V} \mathbf{j}(\mathbf{r}, t') \cdot \mathbf{e}(\mathbf{r}, t') dV dt' \geq 0 \tag{2}$$

for bound charge carriers in passive material that can be lossy as well as lossless. We will refer to a material that satisfies the inequality in (2) for all V as being *unconditionally passive*. The current $\mathbf{j}(\mathbf{r}, t)$ in (2) can include conduction current as well as the current produced by the local motion of the bound charges if the changes in the kinetic and potential energy transferred by the drift velocity of the conduction charges are negligible compared with the out-of-band heat energy produced by the conduction current (as is normally the case).

3 Poynting's Theorem for Dipolar Continua

It can be proven that the total electromagnetic power at time t entering a closed surface S lying in free space is given by the integral over S of the microscopic Poynting vector, namely

$$P(t) = -\int_{S} \hat{\mathbf{n}} \cdot [\mathbf{e}(\mathbf{r}, t) \times \mathbf{h}(\mathbf{r}, t)] dS \tag{3}$$

where $\hat{\mathbf{n}}$ is the unit normal directed away from the volume V enclosed by S, and the microscopic magnetic fields are related by $\mathbf{b} = \mu_0 \mathbf{h}$ because S is in free-space.

From Maxwell's equations in an ideal dipolar continuum, it follows that the components of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$, tangential to an interface between free space and the polarized material are continuous, provided there are no equivalent polarization surface currents at the interface of the polarized material, that is, no delta functions in the electric and magnetic polarization densities, $\mathbf{P}$ or $\mathbf{M}$, at the interface [5]. Such delta functions can occur for polarization with constitutive parameters that are strongly spatially dispersive [1] and even for spatially nondispersive material characterized by extreme anisotropic constitutive parameters with the values of some elements approaching zero and others infinity at the interface [5]. If we avoid these pathologically extreme constitutive parameters and also assume low enough

frequencies that spatial dispersion is negligible, then the tangential components of the ideal-continuum $\mathbf{E}$ and $\mathbf{H}$ are continuous across the free-space/dipolar-material interface. This implies that if we remove an infinitesimally thin shell of the ideal continuum containing S (everywhere that the closed surface S passes through the dipolar continuum), the value of $\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})$ will be continuous across the resulting free-space shell. Moreover, since the continuum fields equal the microscopic fields in the free-space shell, that is

$$\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H}) = \hat{\mathbf{n}} \cdot (\mathbf{e} \times \mathbf{h}) \tag{4}$$

in the free-space shell, one finds from (3) that the total instantaneous power flowing across a closed surface S in an ideal dipolar continuum is given by the integration of the continuum Poynting vector $P(t) = -\int_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)]dS$, or with the aid of Maxwell's ideal-continuum dipolar equations

$$P(t) = -\int\limits_S \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)]dS = \int\limits_V \left[\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \cdot \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \cdot \mathbf{H}(\mathbf{r},t)\right] dV \,. \tag{5}$$

Thus, we have determined that in an ideal continuum the integral of the continuum Poynting vector over a closed surface S of a volume V exactly equals the instantaneous power flow across that surface, provided the constitutive parameters do not take on extreme enough values and the spatial dispersion is not strong enough over the operational bandwidths to produce equivalent surface polarization currents at a hypothetical interface S between the polarized material and free space. Furthermore, the surface integral of the continuum Poynting vector equals the corresponding volume integral of the fields on the right-hand side of (5).

It should be noted that in some contexts bianisotropic media are considered to be spatially dispersive because the electric and magnetic fields in bianisotropic media are proportional to the curls of the magnetic and electric fields, respectively. However, since this weak spatial dispersion maintains continuous $\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})$ across interfaces between the bianisotropic continua and free space, for the purposes of this work, we can include the weak spatial dispersion of bianisotropic media within the category of spatially nondispersive media.

The foregoing argument that has led to Poynting's theorem in an ideal dipolar continuum and its interpretation in terms of power flow can be applied with a couple of modifications to a macroscopic dipolar continuum comprised of discrete electric and magnetic dipoles. The first modification is that the volume V with surface S to which Poynting's theorem is applied should be no smaller than a macroscopic volume ΔV which is electrically small but still contains so many dipoles that the averaging of the fields over ΔV gives to a good approximation the continuum fields satisfying Maxwell's equations.

The second modification involves a transition layer of finite thickness at an interface between free space and the macroscopic dipolar continuum [1], [6]. Unlike an ideal dipolar continuum, the tangential $\mathbf{E}$ and $\mathbf{H}$ fields in a macroscopic dipolar continuum are not perfectly continuous across an interface but are nearly continuous across a transition layer of thickness δ containing the interface. Fortunately, under the conditions that the discrete dipolar material behaves as a continuum, analytical and numerical results with discrete dipolar arrays indicate that the thickness δ of the transition layer is on the order of the average separation distance d of the dipoles [7], [8]. This means that the infinitesimal thickness of the free-space shell about S that was chosen to derive the above results for the Poynting vectors for the ideal continuum can be made approximately equal to the thickness δ of the transition layer without appreciably changing the average fields within the volume V. Consequently, the equation (4) holds to a good approximation for S at the center of the δ-thick free-space shell that removes a discrete number of dipoles; see Fig. 1. The better the discrete distribution of dipoles approximates a continuum, the thinner the transition layer becomes ($\delta \to 0$) and

the more accurate is the approximation for the macroscopic continuum. The only precedent that we could find for this distinction between ideal and macroscopic continua was in Maxwell's original Treatise. This discovery became one of the highlights of a 2014 paper invited to a special issue of the journal, Progress in Electromagnetics Research, commemorating 150 years of Maxwell's equations [2], [3].

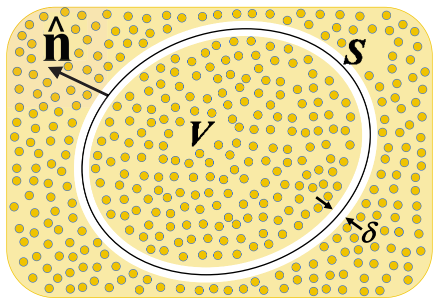

Figure 1: Volume V with its surface S centered in a free-space shell of thickness δ enclosing a large number of discrete electric and magnetic dipoles.

The most important aspect of (5) is that both the surface and volume integrals in (5) are equal to the instantaneous power flow $P(t)$ through the closed surface S into the volume V, exactly for the ideal continuum and approximately for the macroscopic continuum, respectively. Although free-space shells are invoked to prove (5), the fields in (5) are the continuum fields and thus (5) holds exactly within the ideal continuum, and approximately in the macroscopic continuum since $\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})$ is continuous across a free-space/dipolar-material interface under the conditions specified above and satisfied by most dipolar metamaterial continua as well as natural dipolar material continua.

4 Energy Relations for Dipolar Continua

If the volume V is chosen to be a macroscopic volume ΔV that is electrically small but large enough to contain a great number of discrete dipoles, then (5) becomes

$$\Delta P(t) \approx -\int\limits_{\Delta S} \hat{\mathbf{n}} \cdot [\mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t)] dS = \int\limits_{\Delta V} \left[\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \cdot \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \cdot \mathbf{H}(\mathbf{r},t) \right] dV \,. \tag{6}$$

We can also express the power $\Delta P(t)$ in terms of the microscopic fields and sources within ΔV. This can be done by reinstating the surface ΔS of ΔV in the middle of a free-space shell of transition layer thickness δ that removes a discrete number of dipoles, as explained in Section 3, where the volume ΔV contains a large number of discrete electric and magnetic dipoles. Since ΔS is now in free space, the average macroscopic $\mathbf{E}$ and $\mathbf{H}$ fields there are equal to the microscopic $\mathbf{e}$ and $\mathbf{h} = \mathbf{b}/\mu_0$ fields, as explained in Section 3. Thus, combining the macroscopic Poynting's theorem in (6) with the corresponding microscopic Poynting's theorem by means of the equality of the macroscopic and microscopic Poynting's vectors in (4), it can be shown that the following relationship and inequality hold for *unconditionally passive* material

$$\int\limits_{t_0}^{t} \left[\frac{\partial \mathbf{P}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r},t') - \frac{\partial \mathbf{B}(\mathbf{r},t')}{\partial t'} \cdot \mathbf{M}(\mathbf{r},t') \right] dt' \approx \frac{1}{\Delta V} \int\limits_{t_0}^{t} \int\limits_{\Delta V} \mathbf{j} \cdot \mathbf{e} dV dt'$$

$$+\frac{1}{2\Delta V}\int_{\Delta V}\left[\epsilon_0|\mathbf{e}^{\rm ins}(\mathbf{r},t)-\mathbf{E}^{\rm ins}(\mathbf{r},t)|^2+\frac{1}{\mu_0}|\mathbf{b}^{\rm ins}(\mathbf{r},t)-\mathbf{B}^{\rm ins}(\mathbf{r},t)|^2\right]dV\geq 0 \tag{7}$$

where the superscripts "ins" refer to the microscopic and macroscopic fields produced only by the microscopic and macroscopic dipolar sources <u>ins</u>ide ΔV, respectively.

A dipolar material is not necessarily unconditionally passive if it contains permanent dipole moments whose magnitudes can be reduced when subject to applied fields to create an effectively active material. However, nearly all electric polarization can be modeled by either initially randomly oriented molecules with permanent electric dipole moments that stay practically fixed in magnitude as they align in the applied field, or by initially zero electric dipole-moment molecules that distort in the applied field to produce induced electric dipole moments [9, p. 464], [10, p. 162]. In the latter case, the electric dipoles have no initial energy from which to draw. In the former case, the energy of formation of the electric dipoles cannot be reduced because the initial separation distance between the equal and opposite charges of each dipole cannot increase to allow their mutual force to do negative work and release part of their initial energy of formation. Thus, these two types of electric dipoles satisfy the criterion of unconditional passivity.

4.1 Macroscopic Polarization Energy for Magnetic Dipoles

Since magnetic charge does not exist, magnetic dipoles are created by molecules with circulating currents that can be modeled by perfectly electrically conducting (PEC) wire loops. If the wire loops carry no permanent current so that all the circulating current and magnetic dipole moments are induced by the applied fields, then the distribution of molecules forms a diamagnetic macroscopic continuum whose low-frequency magnetic susceptibility is less than zero.

4.1.1 Macroscopic Polarization Energy for Diamagnetism

In the case of diamagnetism, the initial value of the current and magnetic dipole moment of the wire loops are zero. Therefore, diamagnetic-material continua that can also contain electric polarization produced by electric dipoles with fixed-magnitude or initially zero electric dipole moments exhibit the property of unconditional passivity and thus diamagnetic material satisfies the inequality in (7), specifically

$$\int_{t_0}^{t}\left[\frac{\partial\mathbf{P}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{E}(\mathbf{r},t')-\frac{\partial\mathbf{B}(\mathbf{r},t')}{\partial t'}\cdot\mathbf{M}(\mathbf{r},t')\right]dt'\geq 0\,. \tag{8}$$

As far as we are aware, this is the first consistent, positive semi-definite energy expression that has been derived for macroscopic diamagnetic material.

As a simple example, consider a material with negligible loss and constant real permittivity ϵ and permeability μ over the operational baseband bandwidth $|\omega|<\omega_0$. Then $\mathbf{P}=(\epsilon-\epsilon_0)\mathbf{E}$ and $\mathbf{M}=(1/\mu_0-1/\mu)\mathbf{B}$ over the baseband bandwidth, and (8) reduces to

$$(\epsilon-\epsilon_0)|\mathbf{E}|^2-(1/\mu_0-1/\mu)|\mathbf{B}|^2\geq 0\,. \tag{9}$$

Since $\mathbf{E}$ and $\mathbf{B}$ can be chosen equal to zero independently, this inequality reveals that

$$\epsilon\geq\epsilon_0\,,\quad 0\leq\mu\leq\mu_0 \tag{10}$$

which confirms that the inequality in (8) applies to diamagnetic continua.

5 Application of Energy Relations to Bianisotropic Continua

In this section, the energy theorems summarized immediately above are applied to linear, passive, spatially nondispersive media to obtain frequency-domain expressions for internal energy densities in lossless media and inequalities that the linear constitutive relations must obey in lossless media. The most general linear, spatially nondispersive constitutive relations are those for bianisotropic media and are given in the frequency domain as

$$\mathbf{D}_\omega(\mathbf{r}) = \overline{\boldsymbol{\epsilon}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r}) + \overline{\boldsymbol{\tau}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r}) \tag{11a}$$

$$\mathbf{B}_\omega(\mathbf{r}) = \overline{\boldsymbol{\mu}}(\mathbf{r}) \cdot \mathbf{H}_\omega(\mathbf{r}) + \overline{\boldsymbol{\nu}}(\mathbf{r}) \cdot \mathbf{E}_\omega(\mathbf{r}) \tag{11b}$$

where $\overline{\boldsymbol{\mu}}(\mathbf{r})$, $\overline{\boldsymbol{\epsilon}}(\mathbf{r})$, and $[\overline{\boldsymbol{\nu}}(\mathbf{r}), \overline{\boldsymbol{\tau}}(\mathbf{r})]$ are the permeability dyadic, the permittivity dyadic, and the magneto-electric dyadics, respectively. Like the fields, they are, in general, functions of frequency ω and position $\mathbf{r}$ within the media.

If the magnetic polarization of the bianisotropic material is produced by diamagnetic dipoles, then the inequality in (8) applies. It can be rewritten as

$$\int_{t_0}^{t} \left[\frac{\partial \mathbf{D}(\mathbf{r}, t')}{\partial t'} \cdot \mathbf{E}(\mathbf{r}, t') + \frac{\partial \mathbf{B}(\mathbf{r}, t')}{\partial t'} \cdot \mathbf{H}(\mathbf{r}, t') \right] dt' \geq \frac{1}{2} \left[\epsilon_0 |\mathbf{E}(\mathbf{r}, t)|^2 + \frac{1}{\mu_0} |\mathbf{B}(\mathbf{r}, t)|^2 \right] . \tag{12}$$

Using an analysis similar to the one in [11] and [12] but applied to (12), we find for a lossless frequency window

$$\mathrm{Re} \left\{ \mathbf{E}^*_\omega \cdot (\omega \overline{\boldsymbol{\epsilon}})' \cdot \mathbf{E}_\omega + \mathbf{H}^*_\omega \cdot (\omega \overline{\boldsymbol{\mu}})' \cdot \mathbf{H}_\omega + \mathbf{E}_\omega \cdot \left[(\omega (\overline{\boldsymbol{\nu}}^{\mathrm{T}} + \overline{\boldsymbol{\tau}}^*) \right]' \cdot \mathbf{H}^*_\omega \right\} \geq \left[\epsilon_0 |\mathbf{E}_\omega|^2 + \frac{1}{\mu_0} |\mathbf{B}_\omega|^2 \right] \tag{13}$$

which inplies that

$$[(\omega \epsilon_{ll})' - \epsilon_0] \geq \omega \epsilon'_{ll}/2 \geq 0 \tag{14a}$$

with a similar (though not identical) inequality for the diagonal elements of the permeability dyadic if the orientation of the xyz coordinate system is chosen at each point in space to make the permeability dyadic diagonal (which is always possible because the lossless permeability dyadic is Hermitian), namely

$$[(\omega \mu_{ll})' - \mu_{ll}^2/\mu_0] \geq \omega \mu'_{ll}/2 \geq 0 \, . \tag{14b}$$

Note that as $\omega \to 0$, (14a) implies that

$$\epsilon_{ll}(\omega \to 0) - \epsilon_0 \geq 0 \tag{15}$$

(assuming $\lim_{\omega \to 0}(\omega \epsilon'_{ll}) = 0$). However, (14b) implies that

$$\mu_{ll}(\omega \to 0)[1 - \mu_{ll}(\omega \to 0)/\mu_0] \geq 0 \tag{16}$$

or, equivalently

$$0 \leq \mu_{ll}(\omega \to 0) \leq \mu_0 \tag{17}$$

for diamagnetic material (assuming $\lim_{\omega \to 0}(\omega \mu'_{ll}) = 0$). These inequalities are compatible with diamagnetic material, yet incompatible with ordinary nondiamagnetic (paramagnetic) material.

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